## 2015-2016 Curriculum Blueprint  
**Grade: 12**  
**Course:** Mathematics for College Readiness

### Unit 1: Equations and Inequalities

#### Key Vocabulary:
- Information
- Content limits, Attributes/Stimulus, and additional site that contains the Specifications to include the Specifications
- The below benchmark(s) is linked to the CPALMS

#### Instructional Focus Benchmarks

The below benchmark(s) is linked to the CPALMS site that contains the Specifications to include the Specifications.

#### Benchmarks/Standards

<table>
<thead>
<tr>
<th>Benchmark/Standard</th>
<th>Supporting Florida Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.7.EE.2.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</td>
<td><strong>CFAC Math for College Readiness Test Item Specifications</strong></td>
</tr>
<tr>
<td>MAFS.7.NS.1.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</td>
<td><strong>MAFS.7.NS.1.2:</strong> Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers.</td>
</tr>
<tr>
<td>MAFS.8.EE.1.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.</td>
<td><strong>MAFS 912.A-CED.1.1:</strong> Create equations and inequalities in one variable and use them to solve problems.</td>
</tr>
<tr>
<td>MAFS 912.A-CED.1.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.</td>
<td><strong>MAFS 912.A-REI.11:</strong> Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</td>
</tr>
<tr>
<td>MAFS 912.A-SSE.1.1: Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients.</td>
<td><strong>MAFS 912.A-SSE.1.1:</strong> Interpreting expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms.</td>
</tr>
</tbody>
</table>

#### Learning Goal:

**Students will be able to learn to represent real world problems using mathematical representation involving equations and inequalities.**

#### Objectives:

1. Simplify mathematical expressions using order of operations and laws of exponents.
2. Produce math models to represent real world problems by translating verbal statements into mathematical statements.
3. Utilize formulas to solve simple interest and linear growth and explain the relationship between them.
4. Solve linear inequalities by using inverse operations.
5. Solve and sketch absolute value equations by converting to linear inequalities.

#### Essential Content & Understanding:

**Real Numbers and Number Operations**

1. Perform operations of Real Numbers using order of operations.
2. Use properties of Real Numbers.
3. Simplify exponential expressions and use them in scientific notation conversions.

**Algebraic Expressions and Models**

1. Evaluate expressions.
2. Simplify expressions.
3. Use math models to represent relationships.

**Rewriting Equations and Formulas**

1. Write equations with more than one variable.
2. Rewrite common formulas to show relationships between variables.

**Solving Linear Inequalities**

1. Use equation solving techniques to simplify inequalities.
2. Graph inequalities and test values for shading.
3. Solve and graph compound inequalities

#### Essential Questions:

- How does one evaluate algebraic expressions?
- How can algebra be used to model real world problems?

#### Resources/Links:

- **Supplemental Resources:**
  - Project # 1 – 2 Days
  - College Planning and Application Portfolio Project
  - MCR Portfolio
  - College Admissions Parent Letter
  - Precomputer Worksheet
  - Postcomputer Worksheet
  - Request for Letter of Recommendation Form
  - Reflection Sheet

- **Writing Links:**
  - Higher Order Questioning

- **Remediation & Enrichment Resources**

### Unit 1: Equations and Inequalities

#### Time Allowed:

- Trad – 8 weeks
- 1st Quarter
### Math 12: Mathematics for College Readiness

#### Unit 1: Equations and Inequalities

- Interpreting expressions by viewing one or more of their parts as a single entity.

**MAFS.912.F-BF.1.1:** Write a function that describes a relationship between two quantities. 
  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.  
  b. Combine standard function types using arithmetic operations.  
  c. Compose functions.

**MAFS.912.N-Q.1.1:** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
### Instructional Focus Benchmarks

The below benchmark(s) is linked to the CPALMS site that contains the Specifications to include the Content limits, Attributes/Stimulus, and additional information.

### Key Vocabulary:
1. Domain
2. Range
3. Function
4. Slope-Intercept
5. Linear
6. Intersection
7. Piecewise functions
8. Absolute Value Functions
9. Discontinuity

### Learning Goal:
Students will be able to create and graph equations of lines using charts and equations.

### Objectives:
1. Generate tables and graphs to evaluate linear functions.
2. Write linear equations and inequalities.
3. Sketch graphs for piecewise functions and absolute value functions with and without calculators.

### Essential Content & Understanding:

#### Linear Equations and Graphs
1. Generate a table of values
2. Calculate the slope of a line
3. Write linear equations and inequalities
4. Graph linear equation and inequalities
5. Solve real-world problems involving relations and functions

#### Piecewise Functions
1. Graph with a graphing calculator.
2. Create a table for graphing piecewise functions
3. Graph piecewise functions
4. Identify key features
   a. Domain and range
   b. Discontinuities

#### Absolute Value Functions
1. Graph with a graphing calculator.
2. Create a table for graphing absolute value functions
3. Graph absolute value functions
4. Identify key features
   a. Domain and range
   b. Kinks

#### Translations and Composition of Functions
1. Graph with a graphing calculator.
2. Perform operations with functions
3. Evaluate compositions of functions

### Essential Questions:
- How does one identify the domain and range for mathematical functions?
- How does one calculate a table of values for a given function?
- What key features can be anticipated in the graphs of piecewise and absolute value functions?

### Resources/Links:
- **Supplemental Resources:**
  - [Project # 2](#)
  - [College Experience Interview](#)

- **Writing Links:**
  - [Higher Order Questioning](#)

- **Remediation & Enrichment Resources**
specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

MAFS.912.F-IF.1.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

MAFS.912.F-IF.2.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

MAFS.912.F-IF.2.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

MAFS.912.F-IF.2.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

MAFS.912.F-IF.3.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. 

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing
**Unit 2: Linear Equations and Functions**

- Intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**MAFS.912.S-ID.2.5:** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

**MAFS.912.G-GPE.2.5:** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

**MAFS.912.G-GPE.2.6:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

**MAFS.912.G-GPE.2.7:** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

**MAFS.912.S-ID.3.7:** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
## Unit 3: Systems of Equations and Inequalities

### Instructional Focus Benchmarks

The below benchmark(s) is linked to the CPALMS site that contains the Specifications to include the Content limits, Attributes/Stimulus, and additional information.

### Key Vocabulary:

1. System of equations
2. Consistent, Dependent
3. Matrix, Scalar
4. Determinant
5. Inverse Matrix

### Benchmarks/Standards

**Supporting Florida Standards**

**CFAC Math for College Readiness Test Item Specifications**

- **MAFS.912.A-CED.1.3**: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- **MAFS.912.A-REI.3.5**: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- **MAFS.912.A-REI.3.6**: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- **MAFS.912.A-REI.4.11**: Explain why the x-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- **MAFS.912.F-IF.3.7**: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-continuous functions from their equations.

### Learning Goal:

Students will be able to solve linear systems and solve problems involving linear equations and linear inequalities.

### Objectives:

1. Determine solutions for linear systems by graphing with and without calculators.
2. Determine solutions for linear systems by algebraically.
3. Determine solutions for linear inequality systems by graphing with and without calculators.
4. Solve non-linear systems by graphing.

### Essential Content & Understanding:

#### Linear Systems by Graphing

1. Graph to determine intersections using:
   a. \( x \) & \( y \) intercepts
   b. Tables of values
   c. Slope-Intercept equation
   d. Graphing calculators

#### Linear Systems Algebraically

1. Solve using substitution method
2. Solve using elimination
   a. Addition and subtraction
   b. Multiplication

#### Solve Non-Linear Systems

1. Overview using graphing calculators

### Essential Questions:

- What is meant by the solution of a linear system?
- What types of problems can be solved using linear systems?
- How can a graphing calculator be used to solve systems of equations and non-linear systems?

### Resources/Links:

**Supplemental Resources:**

**Writing Links:**

**Higher Order Questioning**

**Remediation & Enrichment Resources**
| defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. |  |  |  |
# Unit 4: Polynomials and Polynomial Functions

## Instructional Focus Benchmarks

The below benchmark(s) is linked to the CPALMS site that contains the Specifications to include the Content limits, Attributes/Stimulus, and additional information.

**Key Vocabulary:**
1. Polynomial Function
2. Remainder Theorem
3. Factor Theorem
4. Rational Zero Theorem
5. Descartes Rule of Signs
6. Fundamental Theorem of Algebra

## Learning Goal:

Students will be able to evaluate, analyze and graph polynomial functions.

## Objectives:

1. Evaluate, analyze, and graph a polynomial function using t-tables, transformations, and technology.
2. Use factoring to solve polynomial equations.
3. Divide polynomials and relate the result to the remainder and factor theorems.
4. Utilize the Fundamental Theorem of Algebra to determine the number of zeros, and find the rational zeros of a polynomial using Descartes Rule of Signs.

## Benchmarks/Standards

**Supporting Florida Standards**

- **MAFS.912.A-APR.1.1:** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **MAFS.912.A-APR.2.3:** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **MAFS.912.A-SSE.1.2:** Use the structure of an expression to identify ways to rewrite it.
- **MAFS.912.A-SSE.2.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 
  a. Factor a quadratic expression to reveal the zeros of the function it defines. 
  b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. 
  c. Use the properties of exponents to transform expressions for exponential functions.
- **MAFS.912.F-IF.2.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

## Essential Content & Understanding:

**Perform operations on Polynomials**
1. Addition and subtraction
2. Multiplication and division

**Polynomial Functions**
1. Factor and solve Polynomial Functions
2. Divide Polynomial Functions

**Find Rational Zeros**
1. Overview with graphing calculator.
2. Use the Remainder and Factor Theorem
3. Use the Fundamental Theorem of Algebra
4. Overview with graphing calculator.

**Graph polynomial functions**

## Essential Questions:

- How can synthetic substitution be used to find the value of a polynomial function?
- What does the end behavior tell you about the graph of a polynomial function?
- What information is provided by the factors of a polynomial function?
- What is the Fundamental Theorem of Algebra?

## Resources/Links:

**Supplemental Resources:**

**Writing Links:**

**Higher Order Questioning**

**Remediation & Enrichment Resources**
Unit 4: Polynomials and Polynomial Functions

- negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

MAFS.912.F-IF.3.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
## Unit 5: Rational Equations and Functions

### Learning Goal:
Students will be able to perform operations on and graph rational equations in real world applications.

### Objectives:
1. Determine direct, inverse, and joint variations using formulas.
2. Graph rational functions using tables, transformations, and technology.
3. Use factoring to simplify rational expressions and solve rational equations.

### Key Vocabulary:
1. Rational Functions
2. Reciprocal
3. Complex Fraction
4. Vertical & Horizontal Asymptotes

### Instructional Focus Benchmarks
The below benchmark(s) is linked to the CPALMS site that contains the Specifications to include the Content limits, Attributes/Stimulus, and additional information.

### Benchmarks/Standards

#### Supporting Florida Standards

<table>
<thead>
<tr>
<th>CFAC Math for College Readiness Test Item Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.8.NS.1.1: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.</td>
</tr>
<tr>
<td>MAFS.8.NS.1.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.</td>
</tr>
</tbody>
</table>
| MAFS.912.A-APR.4.6: Rewrite simple rational expressions in different forms; write 
\[ \frac{a(x)}{b(x)} \] 
 in the form 
\[ q(x) + \frac{r(x)}{b(x)} \]
where \( a(x), b(x), q(x), \) and \( r(x) \) are polynomials with the degree of \( r(x) \) less than the degree of \( b(x) \), using inspection, long division, or, for the more complicated examples, a computer algebra system. |
| MAFS.912.A-APR.4.7: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. |
| MAFS.912.A-REI.1.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. |

### Essential Content & Understanding:

#### Evaluate and Apply Direct, Inverse, and Joint Variations

- Work with rational expressions
- Use rational expressions in real life

#### Multiply and Divide Rational Expressions

- Work with rational expressions
- Use rational expressions in real life
- Simplify complex fractions

#### Solve Rational Equations

- Solve a rational equation
- Use rational expressions in real life

#### Graph Rational Functions

- Graphing a rational function
- Using rational functions in real life

#### Applications of Rational Functions

- Using rational functions in real life

### Essential Questions:
- What is the graph of a rational function called? What special characteristics occur in these graphs?
- What types of real life problems can be modeled by rational functions?
- Since rational expressions involve fractions, how do we go about adding, subtracting, multiplying, and dividing them?
- What is the best way to solve a rational equation? What is important to keep in mind when doing so?

### Resources/Links:

#### Supplemental Resources:

- Writing Links
  - Higher Order Questioning
- Remediation & Enrichment Resources
### 2015-2016 Curriculum Blueprint  
**Grade: 12**  
**Course:** Mathematics for College Readiness

#### Unit 5: Rational Equations and Functions

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
</table>
| MAFS.912.A-SSE.2.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  
  a. Factor a quadratic expression to reveal the zeros of the function it defines.  
  b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  
  c. Use the properties of exponents to transform expressions for exponential functions. |
| MAFS.912.F-IF.2.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |
| MAFS.912.F-IF.3.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.  
  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.  
  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  
  c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.  
  d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.  
  e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. |
## 2015-2016 Curriculum Blueprint  
### Grade: 12  
### Course: Mathematics for College Readiness

## Unit 6: Powers, Roots, and Radicals

<table>
<thead>
<tr>
<th>Instructional Focus Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>The below benchmark(s) is linked to the CPAIMS site that contains the Specifications to include the Content limits, Attributes/Stimulus, and additional information.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key Vocabulary:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rational Exponent</td>
</tr>
<tr>
<td>2. nth Root</td>
</tr>
<tr>
<td>3. cube roots</td>
</tr>
</tbody>
</table>

### Learning Goal:
Students will be able to solve expressions and equations using properties of exponents.

### Objectives:
1. Use the properties of nth roots, powers, and radicals to simplify radicals and equations with radicals including real-life application problems.
2. Graph square and cube root functions using t-tables, transformations, and technology.
3. Solve equations involving radicals.

### Benchmarks/Standards  
**Supporting Florida Standards**
- **CFAC Math for College Readiness Test Item Specifications**
  - MAFS.8.EE.1.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions.
  - MAFS.912.A-SSE.2.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions.
  - MAFS.912.F-BF.2.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
  - MAFS.912.F-BF.2.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
  - MAFS.912.F-IF.3.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show

### Essential Content & Understanding:
- **nth Roots and Rational Exponents**
  1. Convert between rational exponents and radicals
  
- **Properties of Rational Exponents**
  1. Simplify using rules for addition and subtraction of exponents
  2. Simplify using rules for multiplication of exponents

- **Power Functions and Function Operations**
  1. Solve a rational equation
  2. Use rational expressions in real life

- **Graph Square Root & Cube Root Functions**
  1. Graphing a rational function
  2. Using rational functions in real life

- **Solve radical equations**
- **Perform operations with complex numbers**
- **College Planning and Scheduling Project**

### Essential Questions:
- What is the relationship between powers and radicals?
- What are the rules that govern exponents and radicals?
- How does one perform operations involving functions that contain powers or radicals?

### Resources/Links:
- **Supplemental Resources:**
  - Project # 3 – 3 Days
  - College Planning and Scheduling Project – Part A
  - College Planning and Scheduling Project – Part B
- **Writing Links:**
  - Higher Order Questioning

### Remediation & Enrichment Resources
<table>
<thead>
<tr>
<th>Unit 6: Powers, Roots, and Radicals</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>intercepts, maxima, and minima.</strong></td>
</tr>
<tr>
<td>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</td>
</tr>
<tr>
<td>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</td>
</tr>
<tr>
<td>d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</td>
</tr>
<tr>
<td>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</td>
</tr>
<tr>
<td><strong>MAFS.912.F-IF.3.8:</strong> Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</td>
</tr>
<tr>
<td>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</td>
</tr>
<tr>
<td>b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions and classify them as representing exponential growth or decay.</td>
</tr>
<tr>
<td><strong>MAFS.912.N-RN.1.1:</strong> Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.</td>
</tr>
<tr>
<td><strong>MAFS.912.N-RN.1.2:</strong> Rewrite expressions involving radicals and rational exponents using the properties of exponents.</td>
</tr>
<tr>
<td><strong>MAFS.912.N-RN.2.3:</strong> Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</td>
</tr>
</tbody>
</table>
# Unit 7: Quadratic Functions

## Instructional Focus Benchmarks
The below benchmark(s) is linked to the CPALMS site that contains the Specifications to include the Content limits, Attributes/Stimulus, and additional information.

### Key Vocabulary:
1. Quadratic Function
2. Complex Number
3. Pure Imaginary Numbers
4. Imaginary Unit
5. Complex Conjugates
6. Discriminant

## Learning Goal:
Students will be able to solve and graph quadratic equations involving complex numbers.

### Objectives:
1. Calculate sums, differences, products, and quotients for complex numbers by combining like terms.
2. Solve and graph quadratic equations with complex solutions by completing the square and the Quadratic Formula.

## Benchmarks/Standards
Supporting Florida Standards

**CFAC Math for College Readiness Test Item Specifications**

- **MAFS.912.A-REI.2.4:** Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.
- **MAFS.912.A-SSE.1.2:** Use the structure of an expression to identify ways to rewrite it.
- **MAFS.912.A-SSE.2.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions.
- **MAFS.912.F-IF.3.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and

## Essential Content & Understanding:

**Graph quadratic functions**

**Approximate Solutions of quadratic equations using technology**

**Solve quadratic equations**
1. By factoring
2. By completing the square
3. Solve a rational equation
4. Use rational expressions in real life

**Complex Numbers**
1. Simplify and graph complex numbers
2. Perform operations on complex numbers
   a. Addition and subtraction
   b. Multiplication and division
3. Find the absolute value of complex numbers

**The quadratic formula and discriminant**
1. Solving equations with the quadratic formula
2. Solve using the quadratic formula in real-world problems
3. Solve quadratic equations with complex solutions

**Solve real-world applications involving quadratic equations**

## Essential Questions:
- How are complex numbers defined?
- How does one perform operations with complex numbers?
- What types of equations may have complex solutions?

## Resources/Links:
**Supplemental Resources:**

**Writing Links:**

**Higher Order Questioning**

**Remediation & Enrichment Resources**
absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**MAFS.912.F-IF.3.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions and classify them as representing exponential growth or decay.

**MAFS.912.S-ID.2.6.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.
### Instructional Focus Benchmarks
The below benchmark(s) is linked to the CPALMS site that contains the Specifications to include the Content limits, Attributes/Stimulus, and additional information.

### Key Vocabulary:
1. Inverse Function
2. Exponential Function
3. Logarithmic Function
4. Compound Interest
5. Natural Logarithm Common
6. Logarithm
7. Natural Base e
8. Change of Base

### Learning Goal:
Students will be able to solve exponential and logarithmic equations.

### Objectives:
1. Graph inverse, exponential, and logarithmic functions using tables, transformations, and technology.
2. Define and use properties of logarithms to simplify and solve logarithmic expressions and equations including compound interest and exponential growth and decay problems.

### Benchmarks/Standards

#### Supporting Florida Standards
CFAC Math for College Readiness Test Item Specifications
MAFS.912.A-SSE.1.1: Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
MAFS.912.F-BF.1.1: Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. c. Compose functions.
MAFS.912.F-BF.2.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
MAFS.912.F-IF.2.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

#### Essential Content & Understanding:

<table>
<thead>
<tr>
<th>Topic</th>
<th>Overview</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exponential Growth</strong></td>
<td>- Brief Overview</td>
</tr>
<tr>
<td></td>
<td>1. Graph exponential growth functions</td>
</tr>
<tr>
<td></td>
<td>2. Use exponential growth functions</td>
</tr>
<tr>
<td><strong>Exponential Decay</strong></td>
<td>- Brief Overview</td>
</tr>
<tr>
<td></td>
<td>1. Graph exponential decay functions</td>
</tr>
<tr>
<td></td>
<td>2. Use exponential decay functions</td>
</tr>
<tr>
<td><strong>The Number e</strong></td>
<td>- Brief Overview</td>
</tr>
<tr>
<td></td>
<td>1. Use natural base e</td>
</tr>
<tr>
<td></td>
<td>2. Use e in real world applications</td>
</tr>
<tr>
<td><strong>Logarithmic Functions</strong></td>
<td>- Brief Overview</td>
</tr>
<tr>
<td></td>
<td>1. Evaluate logarithmic functions</td>
</tr>
<tr>
<td></td>
<td>2. Graph logarithmic functions</td>
</tr>
<tr>
<td><strong>Properties of Logarithms</strong></td>
<td>- Brief Overview</td>
</tr>
<tr>
<td></td>
<td>1. Use properties of logarithms</td>
</tr>
<tr>
<td><strong>Solving Exponential and Logarithmic Equations</strong></td>
<td>- Brief Overview</td>
</tr>
<tr>
<td></td>
<td>1. Solving exponential equations</td>
</tr>
<tr>
<td></td>
<td>2. Solving logarithmic equations</td>
</tr>
</tbody>
</table>

### Essential Questions:
- How is the exponential growth model related to the compound interest formula?
- How can the exponential decay model be used to predict the decline of record sales?
- What is the number e? Where does it occur? How does it relate to the compound interest formula?
- What is a logarithmic function and how is it related to an exponential function?
- How are the properties of logarithms similar to the rules for working with exponents?
- How can we use the properties of exponents and logarithms to solve those types of equations?

### Resources/Links:

#### Supplemental Resources:

#### Writing Links:

#### Higher Order Questioning

#### Remediation & Enrichment Resources
Unit 8: Exponential and Logarithm Functions

MAFS.912.F-IF.3.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.  
  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.  
  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  
  c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.  
  d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.  
  e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

MAFS.912.F-IF.3.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.  
  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.  
  b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions and classify them as representing exponential growth or decay.
### Instructional Focus Benchmarks

The below benchmark(s) is linked to the CPALMS site that contains the Specifications to include the Content limits, Attributes/Stimulus, and additional information.

### Key Vocabulary:

### Learning Goal:
Students will be able to represent and interpret data using statistics and algebraic modeling.

### Objectives:
1. Determine the mean, median, and mode from a given list of numbers.
2. Demonstrate accuracy and precision solving real number operations.
3. Review additional algebra topics such as quadratic functions.
4. Learn and implement test taking strategies.

### Benchmarks/Standards

#### Supporting Florida Standards

<table>
<thead>
<tr>
<th>Benchmark/Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFAC Math for College Readiness Test Item Specifications</td>
</tr>
<tr>
<td>MAPS.912.S-ID.2.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</td>
</tr>
<tr>
<td>MAPS.912.G-GPE.2.5: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).</td>
</tr>
<tr>
<td>MAPS.912.G-GPE.2.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.</td>
</tr>
<tr>
<td>MAPS.912.G-GPE.2.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.</td>
</tr>
<tr>
<td>MAPS.912.N-Q.1.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</td>
</tr>
<tr>
<td>MAPS.912.N-Q.1.2: Define appropriate quantities for the purpose of descriptive modeling.</td>
</tr>
</tbody>
</table>

### Essential Content & Understanding:

- PERT test review
- Review statistics – Measures of Central Tendency
- Perform operations on real numbers without calculators
- Review Basic Algebra Concepts
  - a. Identify elements of number sets
  - b. Write inequalities using interval notation
  - c. Review all aspects of graphing a linear function
  - d. Review factoring
  - e. Extended practice solving radical problems using all basic operations
- Extended Practice on MAT 1033 Objectives

### Essential Questions:
- How are statistics used to represent and misrepresent information?
- How has your confidence level about taking the PERT changed?

### Resources/Links:

#### Supplemental Resources:

- Practice Exams
  - A – Basic Algebra II
  - B – Florida College Basic Skills
  - C – Accuplacer Questions
  - D – Math 1033 Exam Review

#### Writing Links:

- Higher Order Questioning

#### Remediation & Enrichment Resources
MAFS.912.N-Q.1.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

MAFS.912.S-ID.2.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.

MAFS.912.S-ID.3.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
Supporting Florida Standards

MAFS.K12.MP.1.1: Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze given, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MAFS.K12.MP.2.1: Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MAFS.K12.MP.3.1: Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MAFS.K12.MP.4.1: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a
Supporting Florida Standards

practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MAFS.K12.MP.5.1: Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MAFS.K12.MP.6.1: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MAFS.K12.MP.7.1: Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

MAFS.K12.MP.8.1: Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x -
1)(x² + x + 1), and (x - 1)(x³ + x² + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

English Language Development Standards:

ELD.K12.ELL.MA.1 English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

ELD.K12.ELL.SL.1 English language learners communicate for social and instructional purposes within the school setting.