LESSON 9.5 Solving Volume Problems

ESSENTIAL QUESTION
How do you find the volume of a figure made of cubes and prisms?

Volume of a Triangular Prism
The formula for the volume of a rectangular prism can be used for any prism.

Volume of a Prism
The volume $V$ of a prism is the area of its base $B$ times its height $h$.

$$V = Bh$$

EXAMPLE 1
7.G.2.6
Bradley's tent is in the shape of a triangular prism. How many cubic feet of space are in his tent?

STEP 1 Find the base area $B$ of the triangular prism.

$$B = \frac{1}{2}bh$$

Area of a triangle with base length $b$ and height $h$

Substitute 6 for $b$ and 4 for $h$.

$$= \frac{1}{2}(6)(4)$$

$$= 12 \text{ ft}^2$$

STEP 2 Find the volume of the prism.

$$V = Bh$$

Volume of a prism with base area $B$ and height $h$

Substitute 12 for $B$ and 9 for $h$.

$$= (12)(9)$$

$$= 108 \text{ ft}^3$$

The volume of Bradley's tent is 108 ft$^3$.

Reflect

1. Analyze Relationships For a prism that is not a rectangular prism, how do you determine which sides are the bases?

YOUR TURN

2. Find the volume of the prism.

$$= \frac{1}{2}(7)(24)(22)$$
Volume of a Trapezoidal Prism

Prisms are named for the polygons that form their bases. In this lesson, you will focus on prisms whose bases are either triangles or quadrilaterals other than squares and rectangles.

EXAMPLE 2

Cherise is setting up her tent. Her tent is in the shape of a trapezoidal prism. How many cubic feet of space are in her tent?

STEP 1

Find the base area $B$ of the trapezoidal prism.

$$B = \frac{1}{2}(b_1 + b_2)h$$

where $b_1$ and $b_2$ are the bases and $h$ is the height.

Substitute 6 for $b_1$, 4 for $b_2$, and 4 for $h$.

$$B = \frac{1}{2}(6 + 4) \times 4 = 10 \times 4 = 20 \text{ ft}^2$$

STEP 2

Find the volume of the prism.

$$V = Bh$$

where $B$ is the base area and $h$ is the height.

Substitute 20 for $B$ and 9 for $h$.

$$V = 20 \times 9 = 180 \text{ ft}^3$$

The volume of Cherise’s tent is $180 \text{ ft}^3$.

Reflect

3. Look for a Pattern  How could you double the volume of the tent by doubling just one of its dimensions?

__________________________________________________________________________

4. What If?  How would doubling all the dimensions of the prism affect the volume of the tent?

__________________________________________________________________________

YOUR TURN

5. Find the volume of the prism.

__________________________________________________________________________

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Volume of a Composite Solid

You can use the formula for the volume of a prism to find the volume of a composite figure that is made up of prisms.

**EXAMPLE 3**

Allie has two aquariums connected by a small square prism. Find the volume of the double aquarium.

**STEP 1**

Find the volume of each of the larger aquariums.

- \[ V = Bh \] (Volume of a prism)
- \[ = (12)(3) \] (Substitute 3 × 4 = 12 for \( B \) and 3 for \( h \).)
- \[ = 36 \text{ ft}^3 \]

**STEP 2**

Find the volume of the connecting prism.

- \[ V = Bh \] (Volume of a prism)
- \[ = (1)(2) \] (Substitute 1 × 1 = 1 for \( B \) and 2 for \( h \).)
- \[ = 2 \text{ ft}^3 \]

**STEP 3**

Add the volumes of the three parts of the aquarium.

- \[ V = 36 + 36 + 2 = 74 \text{ ft}^3 \]

The volume of the aquarium is 74 ft³.

**Reflect**

6. **What If?** Find the volume of one of the large aquariums on either end using another pair of opposite sides as the bases. Do you still get the same volume? Explain.

**YOUR TURN**

7. The figure is composed of a rectangular prism and a triangular prism. Find the volume of the figure.

\[ \text{Volume of rectangular prism} = \text{length} \times \text{width} \times \text{height} \]

\[ \text{Volume of triangular prism} = \frac{1}{2} \times \text{base} \times \text{height} \]

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1. Find the volume of the triangular prism. (Example 1)
   \[ V = \frac{1}{2}bh = \frac{1}{2}(8)(3) = 12 \text{ ft}^3 \]

2. Find the volume of the trapezoidal prism. (Example 2)
   \[ V = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(15 + 5)(3) = 30 \text{ m}^3 \]

3. Find the volume of the composite figure. (Example 3)
   Volume of rectangular prism = 
   Volume of triangular prism = 
   Volume of composite figure = 

Find the volume of each figure. (Examples 2 and 3)

4. The figure shows a barn that Mr. Fowler is building for his farm.

5. The figure shows a container, in the shape of a trapezoidal prism, that Pete filled with sand.

ESSENTIAL QUESTION CHECK-IN

6. How do you find the volume of a composite solid formed by two or more prisms?

   ____________________________________________________________________

   ____________________________________________________________________
7. A trap for insects is in the shape of a triangular prism. The area of the base is 3.5 in$^2$ and the height of the prism is 5 in. What is the volume of this trap?

8. Arletta built a cardboard ramp for her little brothers’ toy cars. Identify the shape of the ramp. Then find its volume.

9. Alex made a sketch for a homemade soccer goal he plans to build. The goal will be in the shape of a triangular prism. The legs of the right triangles at the sides of his goal measure 4 ft and 8 ft, and the opening along the front is 24 ft. How much space is contained within this goal?

10. A gift box is in the shape of a trapezoidal prism with base lengths of 7 inches and 5 inches and a height of 4 inches. The height of the gift box is 8 inches. What is the volume of the gift box?

11. **Explain the Error** A student wrote this statement: “A triangular prism has a height of 15 inches and a base area of 20 square inches. The volume of the prism is 300 square inches.” Identify and correct the error.

12. Find the volume of each figure. Round to the nearest hundredth if necessary.

13. **Multi-Step** Josie has 260 cubic centimeters of candle wax. She wants to make a hexagonal prism candle with a base area of 21 square centimeters and a height of 8 centimeters. She also wants to make a triangular prism candle with a height of 14 centimeters. Can the base area of the triangular prism candle be 7 square centimeters? Explain.